Circular Motion and Simple Harmonic Motion

Introduction

This question is about two related topics: Circular motion and simple harmonic motion (SHM). There are six main types of question:

- Motion in a horizontal circle
- Motion is a vertical circle
- SHM (equations)
- Hooke's Law
- SHM with elastic bands
- Simple pendulum

Motion in a horizontal circle

Follow these steps in each case:

- Draw a diagram to find the radius of the circle of motion and the sine and cosine of the angle involved.
- Draw a second diagram showing the forces acting on the particle.
- Draw a third diagram showing the forces, resolved into horizontal and vertical directions.
- The two equations will be:

  (i) Forces up $=$ Forces down

  (ii) Centripetal force $= F_c = m\omega^2 r$ or $\dfrac{mv^2}{r}$

- Use $F_c = m\omega^2 r$ if you are asked to find the constant angular velocity.
- Use $F_c = \dfrac{mv^2}{r}$ if you are asked to find the constant velocity.
- To convert from velocity to angular velocity, use the equation $v = \omega r$.
- There will be questions in which there will be more than one force. In these cases, the centripetal force is the resultant force towards the centre.

Motion in a vertical circle

The velocity, v, is now a variable, not a constant. Follow these steps:

- Draw a diagram showing the forces on the particle at any point (not at the starting point, not at the ending point).
- Resolve all forces into two directions: towards the centre, and perpendicular to the radius.
- The resultant force towards the centre, $F_c = \dfrac{mv^2}{r}$, as before.
- Conservation of energy applies: $mgh_1 + \frac{1}{2}mu^2 = mgh_2 + \frac{1}{2}mv^2$.
- The best thing to do is to get mv^2 in both equations and equalise them. The answer cannot be far away.

SHM (Equations)

The definition of SHM is as follows: SHM takes place when the acceleration of a particle is proportional to its displacement from a fixed point, o, but in the opposite direction.

The equations which govern SHM are as follows:

1. $a = -\omega^2 x$
2. $v^2 = \omega^2\left(A^2 - x^2\right)$
3. $x = A\sin\omega t$ (if the particle starts at the centre).
4. $x = A\cos\omega t$ (if the particle starts at the extreme).
5. $x = A\sin\left(\omega t + \varepsilon\right)$ or $x = A\cos\left(\omega t + \varepsilon\right)$ (if the particle starts somewhere else).
6. $T = \dfrac{2\pi}{\omega}$
7. Maximum velocity $= \omega A$ (as particle passes through o).
8. Maximum acceleration $= \omega^2 A$ (at extreme point).

There are a few key points to watch:

- x and a are always of opposite sign.
- A and ω are taken to be positive constants.

If you are given, say, that $x = A\sin\omega t$ and asked to prove that this particle performs SHM, you must differentiate twice. You will find that $\dfrac{dy}{dx} = \omega A\cos\omega t$ and that $\dfrac{d^2y}{dx^2} = -\omega^2 A\sin\omega t = -\omega^2 x$.
From this we can conclude that SHM is taking place.

Hooke's Law

Hooke's Law states that if an elastic string has natural length l_0 and elastic constant k, and if it is stretched to an actual length of l, then the force F at each end is given by: $F = k\left(l - l_0\right)$.

Questions with elastic bands can involve statics (where the force up = force down and force left = force right), circular motion or SHM.

SHM with elastic bands

In such questions, follow these steps:

1. Establish the position of the centre of oscillation, o. It might be given in the question, or you might have to find the position of zero force to locate it.
2. Let the particle be displaced a distance x from o.
3. Examine the forces when the particle is displaced a distance x from o.
4. Establish that $a = -\omega^2 x$.
5. The amplitude, A, is determined by how far from o the particle starts.
6. Now use the eight formulae to find whatever is required.

The Simple Pendulum

You should know how to establish the formula $T = 2\pi\sqrt{\frac{l}{g}}$ and apply it to particular cases. This equation often turns up is Question 8 also.

Common Mistakes

- Using the “degree mode” instead of the “radian mode” on a calculator when using the trigonometry buttons (especially when looking for time).
- Not knowing the formulae for SHM.
- Treating circular motion as a statics question.
- In elastic band questions, examining the forces for the starting position only.