Hydrostatics

Introduction

This question is about objects in liquids. There are five main types of question:

- Density and relative density
- Pressure, Weight and Thrust
- The Principle of Archimedes
- Buoyancy in liquids
- Statics of immersed objects

Density and relative density

- The density of water $= 1,000\, kg/m^3$
- Relative density $= \dfrac{\text{Density of object}}{\text{Density of water}}$
- Hence, the relative density of water $= 1$
- The relative density of mercury $= 13 \cdot 6$
- The symbol for density is usually ρ
- The symbol for relative density is usually s

Pressure, Weight and Thrust

- Pressure $= h\rho g$, where $h =$ the depth, $\rho =$ density of liquid.
- Weight $= V\rho g$, where $V =$ the volume, $\rho =$ density of liquid.
- Thrust on a horizontal surface $= PA$, where $P =$ the pressure at a point on the surface and A the area of the surface.
- Thrust on a vertical surface has sometimes appeared on the Leaving Cert exam, although the syllabus is confined to "Thrust on a horizontal surface". In case this happens again, you might need the formula for thrust, T on a vertical surface. It is $T = P_c A$, where $P_c =$ the pressure at the centre of gravity, and $A =$ the area of the surface.

The Principle of Archimedes

The Principle of Archimedes: When an object is wholly or partly immersed in a liquid it suffers an upthrust which is equal in magnitude to the weight of the liquid displaced. This upthrust (a force pushing the object vertically upwards) acts through the Centre of Buoyancy: the centre of gravity of the immersed part of the object.

Buoyancy in liquids

There are three formulae for calculating the buoyancy suffered by an immersed object:

1. Buoyancy $= V\rho g$ (where $V =$ the volume of liquid displaced and $\rho =$ the density of the liquid displaced).
2. Buoyancy in water $= \frac{W}{s}$, where W is the weight of the immersed object and s is the relative density.
3. Buoyancy in a liquid $= \frac{s_l W}{s}$, where W is the weight of the immersed object, s is its relative density and s_l is the relative density of the liquid.

Statics of immersed objects

The weight of a piece of uniform rod is proportional to its length. For example, if a rod (of weight W) has length l metres and x metres are under water, then the weight of the wet part is $\frac{x}{l}W$ and its buoyancy is $\frac{x}{ls}W$ (by rule 3, above).

Divide the object into two parts: the wet and the dry. Treat these as though they are separate. Draw a force diagram, showing the weight, tension, reaction and the buoyancy, acting upward through the centre of buoyancy of the wet part. Now, treat the problem like any statics problem: Forces up = Forces down; clockwise moments = Anti-clockwise moments.

Common mistakes

- Mixing up density and relative density.
- Mixing up pressure and thrust. Thrust is a kind of force.
- Confusing mass and weight. Weight is a force.
- Not being able to use the three methods of finding buoyancy.