Relative Velocity

Introduction

This topic involves looking at the world from a different perspective. We are used to the idea of observing, say, two ships as they sail the seas. This way of looking down from above (like Zeus looking down from Mount Olympus) is sometimes called ***the Olympian View***.

When we study relative velocity, we see life differently. We go on board one of the ships. We observe the other ship, as seen from our ship. The origin is no longer fixed, but moves wherever our ship moves. Looking at the situation in this way makes difficult problems easy to solve.

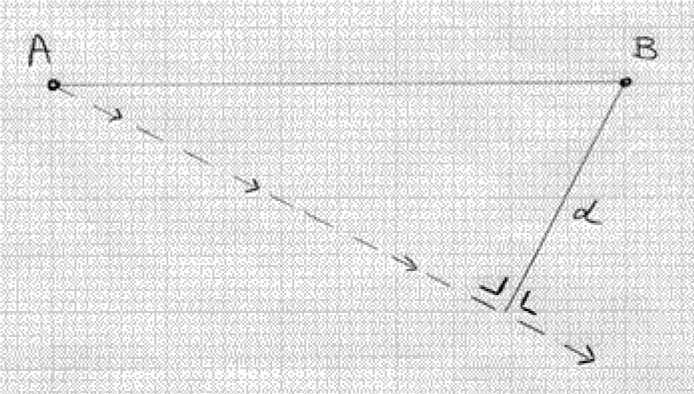
There are four main types of question:

- Problems involving two particles
- Road junctions
- Rivers, currents and winds
- Apparent velocity of the wind

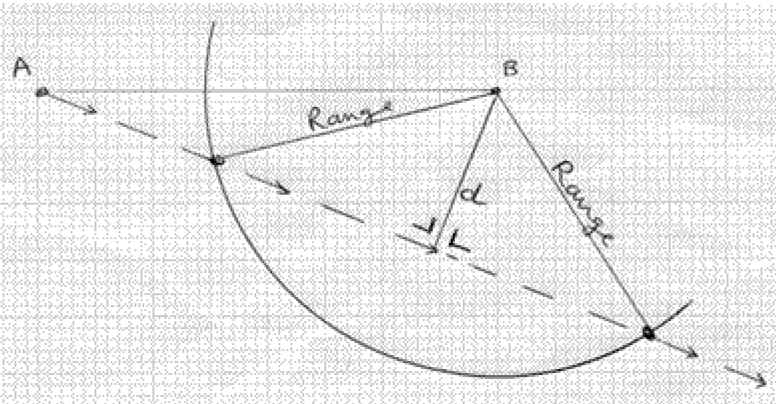
Problems involving two particles

The key formula is $\overrightarrow{v_{ab}} = \overrightarrow{v_a} - \overrightarrow{v_b}$, where v_{ab} means the velocity of A as observed by B (i.e. the velocity of A relative to B).

- To answer this question, you must get away from thinking of problems from the Olympian view and get accustomed to looking at problems relatively. It takes time and practice – be patient.
- Draw plenty of diagrams – and make them clear, large and reasonable accurate.
- If you are told that a car is travelling at $30\,km/h$ in a direction $20°$ North of West, you must learn how to convert this velocity into a vector in terms of $\vec{i}$ and $\vec{j}$. Draw a diagram, and then use your Junior Cert Trigonometry to get the correct answer: it is $-28\cdot19\vec{i} + 10\cdot26\vec{j}\ km/h$.
- You also need to be able to do this process in reverse. Given, say, $\overrightarrow{v_{ab}} = 20\vec{i} + 10\vec{j}\ m/s$, can you find the speed and direction of the particle? Again draw a diagram. The speed $= \sqrt{20^2 + 10^2} = 22\cdot36\,m/s$ and the direction is $\tan^{-1}\left(\frac{10}{20}\right) = 26\cdot565°$ North of East.
- The most common question asked is "Find the shortest distance between the particles". Find the velocity of A relative to B; draw a diagram; show where B is (B stays still, since wherever B goes, the origin goes); now show the path of A relative to B; the shortest distance (d) is found by drawing a straight line from B, perpendicular to the relative path of A.

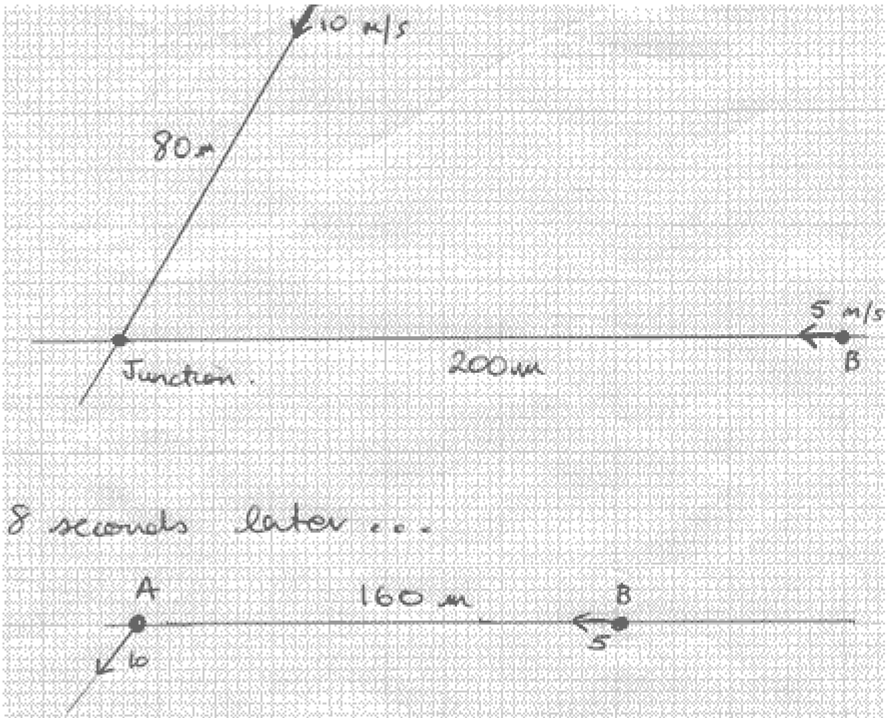


- If you are asked to find for how long are the particles within a certain range of each other, draw a circle around *B* and find the length of the chord (see diagram) using Pythagoras' Theorem. The time taken will be got by $t = \dfrac{\text{distance}}{\text{speed}}$.



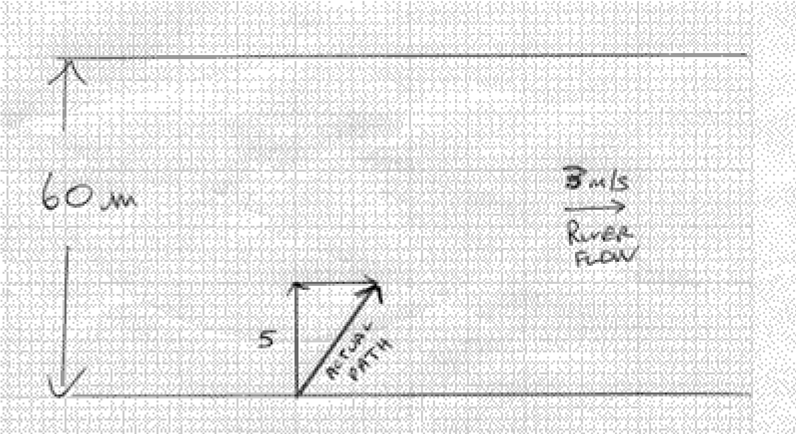
Road junctions

For example let's say you are given the diagram below and asked what is the shortest distance between A and B in subsequent motion. It seems a very difficult problem. But what will happen if you wait 8 seconds until A is at the junction? Now here is the new situation and the problem is much easier. It can be solved using the steps outlined above.

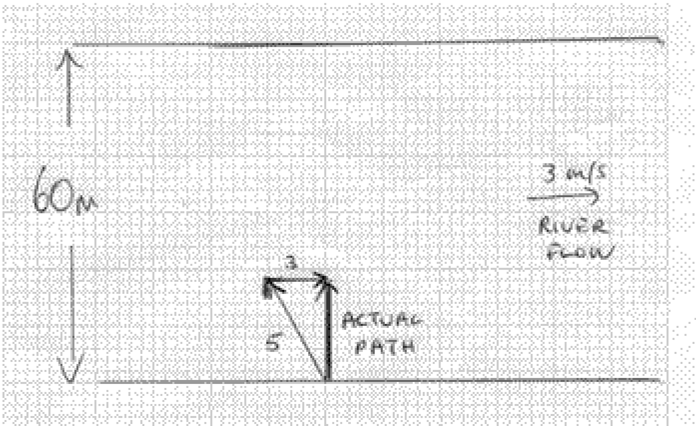


Rivers, currents and winds

If a river is, for example, 60 metres wide and flows with speed $3m/s$. Let's suppose your boat can go at $5m/s$, then remember the two key strategies:



- To cross the river as quickly as possible, head straight across. Every second your boat will be 5 metres closer to the other side, so the time taken will be $\frac{60}{5} = 12$ seconds.



- To cross the river by the shortest path, head upstream, so that you end up going straight across. By Pythagoras' Theorem, you will find that each second brings you 4 m closer to the opposite bank. The time taken will be $\frac{60}{4} = 15$ seconds.

Apparent velocity of the wind

The ***actual*** velocity of the wind and the ***apparent*** velocity of the wind are two different things. For example, if you are running at $5m/s$ and there is a wind straight in your face of $2m/s$, then the wind appears to have a velocity of $7m/s$.

In these problems, let the velocity of the wind be $\overrightarrow{v_w} = x\vec{i} + y\vec{j}$. Write down the given velocity of the person, $\overrightarrow{v_p}$, and find the velocity of the wind relative to the person (how the wind ***appears*** to the person), using the formula $\overrightarrow{v_{wp}} = \overrightarrow{v_w} - \overrightarrow{v_p}$.

Now, if the apparent velocity is from the east or west, the $\vec{j}$-component must equal zero.

If the apparent velocity is from the north or south, the $\vec{i}$-component must equal zero.

If the apparent velocity is from the SW or NE, the $\vec{i}$-component must equal the $\vec{j}$-component.

If the apparent velocity is from the SE or NW, the $\vec{i}$-component must be equal but of opposite sign to the $\vec{j}$-component.

Manufacture the appropriate simultaneous equations and solve.

Common mistakes

- Mixing up the Olympian View with the Relative View.
- Not drawing an appropriate diagram of the relative path.
- Inability to convert a given vector into $\vec{i} - \vec{j}$ form.
- Confusing the actual velocity of the wind with the apparent velocity of the wind.