# Uniform Linear Acceleration

## Introduction

This topic is about particles which move in a straight line and accelerate uniformly. Problems can vary enormously, so you have to have your wits about you. Problems can be broken down into three main categories:

- Constant Uniform Acceleration
- Time-speed graphs
- Problems involving two particles

## Constant Uniform Acceleration

Remember what the following variables represent: $t$=time; $a$=acceleration; $u$=the initial speed; $v$=the final speed; $s$=the displacement from where the particle started. When the acceleration is negative , it is sometimes called a deceleration or retardation. For example, an acceleration of -3 m/s$^2$ is the same as a deceleration (or retardation) of 3 m/s$^2$.

- To answer this question, you will need to use the four key formulae intelligently.

They are: $v = u + at$; $s = \left(\frac{u+v}{2}\right)t$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$.

- It is important to know the second of these equations off by heart; the others appear on page 40 of The Mathematical Tables. Secondly, you may be asked to derive either of the last two equations from the first two. Practice this.
- These four formulae will be useful elsewhere (for example when doing Questions 3 and 4 on projectiles and connected particles).

## Time-speed graphs

Remember that the above formulae may be used only while the acceleration is uniform. If a particle speeds up, but then travels at a constant speed, and then slows down, the above formulae cannot be used for the entire journey. In these cases we solve the problem by drawing a time-speed graph, with time as the horizontal axis.

There are four key points to remember about time-speed graphs:

- The area between the graph and the time-axis represents the distance travelled.
- The slope of the graph represents the acceleration.
- If the particle starts from rest, then $v = at$ [i.e. the final speed will be the product of the acceleration and the time.]
- If a particle accelerates from rest for a time $t_1$ with acceleration $a$ and immediately decelerates to rest in time $t_2$ with deceleration $d$ then $t_1 : t_2 = d : a$.

For example, if the acceleration is $6ms^{-2}$ and the deceleration is $8ms^{-2}$ , then $t_1 : t_2 = d : a = 8 : 6 = 4 : 3$. It follows that $\frac{4}{7}$ of the time will be spent accelerating and $\frac{3}{7}$ of the time will be spent decelerating.

## Problems involving two particles

- If particles $P$ and $Q$ set off together and later overtake each other, then overtaking will occur when $s_P = s_Q$. If, however, $P$ was 25 metres behind $Q$ at the start, then when overtaking occurs, $s_P = s_Q + 25$.
- If $P$ and $Q$ are a distance $l$ apart and move towards each other, they will meet when $s_P + s_Q = l$.
- The greatest gap between particles $P$ and $Q$ occurs when $v_P = v_Q$ (because if their speeds are unequal then the gap is either increasing or decreasing).
- If particle $A$ sets out and, two seconds later, particle $B$ sets out in pursuit, then let $t =$ the time which $A$ spends on the road and let $t - 2 =$ the time which $B$ spends on the road. (Students will often put $t + 2$ instead of $t - 2$).

## Common Mistakes

Common mistakes made in doing this question are

- Assuming that the particle starts from rest, even though this is not stated in the question.
- Using the formulae where they do not apply.
- Jumping into the question before giving it enough clear thought.
- Not drawing a clear time-speed graph.
- Letting $u$ represent the speed at two different moments. For example, if a particle travels from $a$ to $b$, a distance of 30 m in 4 seconds and then travels from $b$ to $c$, a distance of 54 m in a further 3 seconds – how would you find the acceleration? You let $u =$ the speed of the particle at $a$ (not anywhere else!). Then you form an equation for the journey $[ab]$ and another for the journey $[ac]$ . These equations will be (using $s = ut + \frac{1}{2}at^2$ ); $30 = u(4) + \frac{1}{2}a(16)$ and $84 = u(7) + \frac{1}{2}a(49)$. You solve these simultaneous equations to find $a$. NB: You do not form an equation for $[bc]$, as the initial speed will not be the same $u$ as in the first equation.