Leaving Certificate Examination 2007 – Mathematics Ordinary Level – Paper 1

1. (a) $164 \text{ miles} = 164 \times \frac{8}{5} = 262 \cdot 4 \text{ km.}$

(b) (i) $A = 8,500 \times 1 \cdot 04 = €8,840$

(ii) $8,840 \times (1+i) = 9,237 \cdot 80$

$\Rightarrow \quad 8,840 + 8,840i = 9,237 \cdot 80$

$\Rightarrow \quad 8,840i = 397 \cdot 8$

$\Rightarrow \quad i = 0 \cdot 045$

$\Rightarrow$ Interest Rate was $4 \cdot 5\%$ in year 2.

(c) (i) Thursday, Friday & Saturday

$= (9 \times 15 \cdot 60) + (9 \times 15 \cdot 60) + (9 \cdot 5 \times 15 \cdot 60 \times 1 \cdot 5) = €503 \cdot 10$

(ii) Amount earned on Sunday $= 702 - 503 \cdot 10 = 198 \cdot 90$

$15 \cdot 60 \times h \times 1 \cdot 5 = 198 \cdot 90$

$\Rightarrow \quad 23 \cdot 4h = 198 \cdot 9$

$\Rightarrow \quad h = 8 \cdot 5$ hours.

2. (a) $4x - 15 < 1$

$\Rightarrow \quad 4x < 16$

$\Rightarrow \quad x < 4$

$\Rightarrow \quad x = \{0, 1, 2, 3\}$

(b) (i) $$\frac{x+3y+5}{2x+2y} = \frac{\frac{5}{2} + 3\left(\frac{1}{3}\right) + 5}{2\left(\frac{5}{2}\right) + 2\left(\frac{1}{3}\right)} = \frac{\frac{5}{2} + 1 + 5}{5 + \frac{2}{3}} = \frac{15 + 6 + 30}{30 + 4} = \frac{51}{34} = \frac{3}{2}$$

(ii) $2^{x+3} = 4^x \Rightarrow 2^{x+3} = \left(2^2\right)^x \Rightarrow 2^{x+3} = 2^{2x}$

$\Rightarrow \quad x + 3 = 2x \Rightarrow \boxed{x = 3}$

(c) (i) $x - \frac{1}{x} = 2 \Rightarrow x^2 - 2x - 1 = 0$

$$\Rightarrow \quad x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

(ii) Let $x = 1 + \sqrt{2}$

$x - \frac{1}{x} = 2$

$$\Rightarrow \quad 1 + \sqrt{2} - \frac{1}{1+\sqrt{2}} = 2$$

$\Rightarrow \quad \left(1+\sqrt{2}\right)^2 - 1 = 2\left(1+\sqrt{2}\right)$

$\Rightarrow \quad \cancel{1} + 2\sqrt{2} + 2 - \cancel{1} = 2 + 2\sqrt{2}$

$\Rightarrow \quad 2\sqrt{2} + 2 = 2\sqrt{2} + 2$ Q.E.D.

3. (a) $2x = 3(5 - x)$

$\Rightarrow \quad 2x = 15 - 3x$

$\Rightarrow \quad 5x = 15$

$\Rightarrow \quad \boxed{x = 3}$

(b) $\frac{x}{4} - \frac{y}{3} = \frac{5}{6}$ …multiply across by 12

$\Rightarrow \quad 3x - 4y = 10$ …Equation I

Also $\quad 2x - 6 = 3y$

$\Rightarrow \quad 2x - 3y = 6$ …Equation II

$\text{I} \times 2: \quad \cancel{6x} - 8y = 20$

$\text{II} \times (-3): \quad -\cancel{6x} + 9y = -18$ …add

$\Rightarrow \quad \boxed{y = 2} \Rightarrow \boxed{x = 6}$

(c) $f(x) = 2x^3 + 11x^2 + 4x - 5$

(i) $f(-1) = 2(-1)^3 + 11(-1)^2 + 4(-1) - 5 = -2 + 11 - 4 - 5 = 0$ Q.E.D.

(ii) $f(-1) = 0 \Rightarrow (x + 1)$ is a factor.

$$
\begin{array}{r l}
 & 2x^2 + 9x - 5 \\
x + 1 \overline{)} & 2x^3 + 11x^2 + 4x - 5 \\
 & \underline{2x^3 + 2x^2} \quad \text{…subtract} \\
 & 9x^2 + 4x - 5 \\
 & \underline{9x^2 + 9x} \quad \text{…subtract} \\
 & -5x - 5 \\
 & \underline{-5x - 5} \quad \text{…subtract} \\
 & 0
\end{array}
$$

$\therefore \quad 2x^3 + 11x^2 + 4x - 5 = 0$

$\Rightarrow \quad (x + 1)(2x^2 + 9x - 5) = 0$

$\Rightarrow \quad (x + 1)(2x - 1)(x + 5) = 0$

$\Rightarrow \quad \boxed{x = -1}, \boxed{x = \frac{1}{2}}, \boxed{x = -5}$ are the solutions.

4. (a) $3(2-4i)+i(5-6i)$

$= 6-12i+5i+6$

$= 12-7i$

(b) (i) $z=5-3i$

(ii) $|z-1|=|5-3i-1|=|4-3i|=\sqrt{4^2+(-3)^2}=\sqrt{16+9}=\sqrt{25}=5$

(iii) $ki+4z=20$

$\Rightarrow ki+4(5-3i)=20$

$\Rightarrow ki+\cancel{20}-12i=\cancel{20}$

$\Rightarrow (k-12)i=0$

$\Rightarrow k-12=0$

$\Rightarrow \boxed{k=12}$

(c) $u=3+2i$

(i) $u^2+\bar{u}^2=(3+2i)^2+(3-2i)^2=9+\cancel{12i}-4+9-\cancel{12i}-4=10$

(ii) $\dfrac{13}{u}=\dfrac{13}{3+2i}\times\dfrac{3-2i}{3-2i}=\dfrac{39-26i}{9+4}=\dfrac{39-26i}{13}=3-2i=\bar{u}$.

5. (a) $T_n=1-n$

(i) $T_5=1-5=-4$

(ii) $T_5-T_{10}=-4-(1-10)=-4-1+10=5$

(b) $a=3,\quad d=4$

(i) $T_n=a+(n-1)d=3+4(n-1)=3+4n-4=4n-1$

(ii) Let $T_n=200$

$\Rightarrow 4n-1=200$

$\Rightarrow 4n=201$

$\Rightarrow n=50\cdot 25\Rightarrow$ 50 terms are less than 200.

(iii) $S_n=\dfrac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow S_{50}=25\{6+4(49)\}=25\{6+196\}=25\{202\}=5,050$

(c) $\dfrac{1}{3}+\dfrac{1}{9}+\cdots\cdots$

(i) $r=\dfrac{1/9}{1/3}=\dfrac{1}{9}\times\dfrac{3}{1}=\dfrac{1}{3}$

(ii) $S_n=\dfrac{a(1-r^n)}{1-r}=\dfrac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}}=\dfrac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}=\dfrac{\cancel{3}}{2}\left(\dfrac{1}{\cancel{3}}\left(1-\dfrac{1}{3^n}\right)\right)=\dfrac{1}{2}\left(1-\dfrac{1}{3^n}\right)$

(iii) $\dfrac{p}{3}+\dfrac{p}{9}+\cdots\cdots=p\left(\dfrac{1}{3}+\dfrac{1}{9}+\cdots\cdots\right)=p\left(\dfrac{1}{2}\left(1-\dfrac{1}{3^n}\right)\right)=\dfrac{p}{2}\left(1-\dfrac{1}{3^n}\right)$

$S_n=1-\dfrac{1}{3^n}$

$\Rightarrow \dfrac{p}{2}\left(1-\dfrac{1}{3^n}\right)=1-\dfrac{1}{3^n}$ …divide both sides by $1-\dfrac{1}{3^n}$

$\Rightarrow \dfrac{p}{2}=1\Rightarrow \boxed{p=2}$

6. (a) $g(x) = x^2 - 6x$

(i) $g'(x) = 2x - 6$

(ii) $g'(x) = 0$

$\Rightarrow$ $2x - 6 = 0$

$\Rightarrow$ $2x = 6$

$\Rightarrow$ $x = 3$

(b) (i) 6 minutes.

(ii) 5 degrees celcius.

(ii) $C = \frac{1}{2}(t + k)$ …because the relationship is constant, we can simply choose any two corresponding values of C and t.

Let $t = 2, \quad C = -2$

$\Rightarrow$ $-2 = \frac{1}{2}(2 + k)$

$\Rightarrow$ $2 + k = -4$

$\Rightarrow$ $k = -6$

(c) $f(x) = (5x - 2)^4$

(i) $f'(x) = 4(5x - 2)^3(5) = 20(5x - 2)^3$

(ii) $20(5x - 2)^3 = 20$ …divide both sides by 20.

$\Rightarrow$ $(5x - 2)^3 = 1$ …take cube roots on either side.

$\Rightarrow$ $5x - 2 = 1$

$\Rightarrow$ $5x = 3$

$\Rightarrow$ $x = \frac{3}{5}$ …sub this into $f(x)$

$\Rightarrow$ $y = 1$

$\Rightarrow$ Point is $\left(\frac{3}{5}, 1\right)$

7. (a) $y = 6x^4 - 3x^2 + 7x$

$\frac{dy}{dx} = 24x^3 - 6x + 7$

(b) (i) $y = (x^2+9)(4x^3+5)$ …differentiate using the product rule

$\frac{dy}{dx} = (x^2+9)(12x^2) + (4x^3+5)(2x)$

$= 12x^4 + 108x^2 + 8x^4 + 10x$

$= 20x^4 + 108x^2 + 10x$

(ii) $y = \frac{3x}{2x+3}$ …differentiate using the quotient rule.

$\frac{dy}{dx} = \frac{(2x+3)(3)-(3x)(2)}{(2x+3)^2} = \frac{\cancel{6x}+9-\cancel{6x}}{(2x+3)^2} = \frac{9}{(2x+3)^2}$

(c) $s = 2t^2 + 2t$

(i) Speed $= \frac{ds}{dt} = 4t + 2$ …let $t = 2$

$\Rightarrow$ Speed $= 4(2) + 2 = 8 + 2 = 10\, m/s$.

(ii) Acceleration $= \frac{d^2s}{dt^2} = 4\, m/s^2$.

(iii) $s = 2t^2 + 2t = 24$

$\Rightarrow$ $t^2 + t = 12$

$\Rightarrow$ $t^2 + t - 12 = 0$

$\Rightarrow$ $(t+4)(t-3) = 0$

$\Rightarrow$ $t = -4, \quad t = 3$ …only the positive value makes sense

$\Rightarrow$ $t = 3$ seconds.

8. (a) $f(x)=\frac{1}{4}(6-2x)$

$f(5)=\frac{1}{4}(6-10)=\frac{1}{4}(-4)=-1$

(b) $f(x)=x^2-3x$

$f(x+h)=(x+h)^2-3(x+h)$

$f(x+h)-f(x)=(x+h)^2-3(x+h)-x^2+3x$

$=\cancel{x^2}+2xh+h^2-\cancel{3x}-3h-\cancel{x^2}+\cancel{3x}$

$=2xh+h^2-3h$

$\frac{f(x+h)-f(x)}{h}=2x+h-3$

$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h-3)=\boxed{2x-3}$

(c) $f(x)=\frac{1}{x+7}$

(i) $f(k)=1$

$\Rightarrow$ $\frac{1}{k+7}=1$

$\Rightarrow$ $k+7=1$

$\Rightarrow$ $k=-6$

(ii) $f(x)=(x+7)^{-1}$

$f'(x)=-(x+7)^{-2}=\frac{-1}{(x+7)^2}$

(iii) $f'(x)=0$

$\Rightarrow$ $\frac{-1}{(x+7)^2}=0$ …multiply both sides by $(x+7)^2$

$\Rightarrow$ $-1=0$ …impossible

$\Rightarrow$ No turning points.