Leaving Certificate Ordinary Level Mathematics – Paper 2 2007 – Solutions

1. (a) (i) Let the third side be x. Using Pythagoras' Theorem

$x^2 = 10^2 + 24^2$

$\Rightarrow x^2 = 100 + 576$

$\Rightarrow x^2 = 676$

$\Rightarrow x = 26$.

(ii) Perimeter $= 10 + 24 + 26 = 60$ cm.

(b) (i) Length of horizontal line $= 11\cdot 4$ cm.

Length of each offset $= h = 2\cdot 85$ cm.

Length of vertical lines: $y_1 = 0$ cm

$y_2 = 4\cdot 8$ cm

$y_3 = 6\cdot 7$ cm

$y_4 = 3\cdot 8$ cm

$y_5 = 0$ cm.

(ii) Area $= \frac{h}{3}\{\text{First} + \text{Last} + \text{Twice Odd} + \text{Four Times Even ordinates}\}$

$= \frac{2\cdot 85}{3}\{0 + 0 + 2(6\cdot 7) + 4(4\cdot 8 + 3\cdot 8)\}$

$= 0\cdot 95\{13\cdot 4 + 34\cdot 4\}$

$= 0\cdot 95\{47\cdot 8\}$

$= 45\cdot 41\text{cm}^2$

(c) (i) $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10\cdot 5)^2 = 147\pi$

(ii) $V_{sphere} + V_{cylinder} = 147\pi + \pi r^2 h = 147\pi + \pi(10\cdot 5)^2 h = 147\pi + 110\cdot 25\pi h$

$\Rightarrow 147\pi + 110\cdot 25\pi h = 6174\pi$ …divide across by π.

$\Rightarrow 147 + 110\cdot 25h = 6174$

$\Rightarrow 110\cdot 25h = 6027$

$\Rightarrow h = \frac{6027}{110\cdot 25} = 64\cdot 667$ cm.

2. (a) Midpoint $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+6}{2},\frac{-3+9}{2}\right)=(4,3)$

(b) (i) $m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-0}{0+4}=\frac{6}{4}=\frac{3}{2}$

(ii) $y-y_1=m(x-x_1)$

$\Rightarrow$ $y-0=\frac{3}{2}(x+4)$ …multiply across by 2.

$\Rightarrow$ $2y=3x+12$

(iii) Diagram,

(iv) $m_k=-\frac{2}{3}$, $(0,0)\in K$

$\Rightarrow$ $y-0=-\frac{2}{3}(x-0)$

$\Rightarrow$ $3y=-2x$

$\Rightarrow$ $2x+3y=0$ …this is the equation of K.

(c) (i) $(-4,3)\rightarrow(0,0)$

$\Rightarrow$ $(6,-1)\rightarrow(10,-4)=(x_1,y_1)$

$\Rightarrow$ $(2,7)\rightarrow(6,4)=(x_2,y_2)$

$A=\frac{1}{2}|x_1y_2-x_2y_1|=\frac{1}{2}|(10)(4)-(6)(-4)|=\frac{1}{2}|40+24|=\frac{1}{2}(64)=32$.

(ii) $b\rightarrow a=c\rightarrow d$

$\Rightarrow$ $(6,-1)\rightarrow(-4,3)=(2,7)\rightarrow(-8,11)=d$.

3. (a) (i) $x^2 + y^2 = 16$

(ii) $(3,2)$ is inside if distance from $(0,0)$ to $(3,2)$ is less than 4 (the radius).

$d_{(0,0)\to(3,2)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3)^2 + (2)^2} = \sqrt{13} = 3\cdot 6 < 4$

$\Rightarrow$ $(3,2)$ is inside the circle.

(b) (i) $x^2 + y^2 = 10$ $\qquad$ $x - 3y = 0 \Rightarrow x = 3y$

$\Rightarrow$ $(3y)^2 + y^2 = 10$

$\Rightarrow$ $9y^2 + y^2 = 10$

$\Rightarrow$ $10y^2 = 10$

$\Rightarrow$ $y^2 = 1 \Rightarrow y = \pm 1$

If $y = 1$ then $x = 3$

If $y = -1$ then $x = -3$.

$\Rightarrow$ $a(1,3)$, $b(-1,-3)$

(ii) $[ab]$ is diameter if midpoint of $[ab]$ is the centre of the circle $(0,0)$.

$\text{Midpoint}_{[ab]} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1-1}{2}, \frac{3-3}{2}\right) = (0,0)$

$\Rightarrow$ $[ab]$ is a diameter of the circle.

(c) $(x-5)^2 + (y+1)^2 = 34$

(i) $r = \sqrt{34}$, $\qquad c(5,-1)$

(ii) Substitute $(10,-4)$ into the equation:

$(10-5)^2 + (-4+1)^2 = 34$

$\Rightarrow$ $5^2 + (-3)^2 = 34$

$\Rightarrow$ $25 + 9 = 34$

$\Rightarrow$ $34 = 34$ $\qquad$ …satisfies the equation

$\Rightarrow$ $(10,-4)$ is on the circle.

(iii) We need to find the image of the point $(10,-4)$ under central symmetry in the centre $(5,-1)$

$(10,-4) \to (5,-1) \to (0,2)$

$\Rightarrow$ S is a tangent to the circle at the point $(0,2)$.

4. (a) (i) $x = 100°$.

(ii) $y = 80°$.

(b) Given: The triangle abc with $ad \perp bc, be \perp ac$ and $cf \perp ab$.

To Prove: $|bc|.|ad| = |ac|.|be| = |ab|.|cf|$.

Proof: In the triangles adc and bec,

$|\angle adc| = |\angle bec| = 90°$ …given

$\angle acd$ is common to both triangles

$\Rightarrow$ $|\angle cad| = |\angle ebc|$

$\Rightarrow$ the triangles adc and bec are similar

$\Rightarrow$ corresponding sides are in the same ratio

$\Rightarrow$ $\dfrac{|ad|}{|be|} = \dfrac{|ac|}{|bc|}$

$\Rightarrow$ $|bc|.|ad| = |ac|.|be|$

Similarly it may be proved that

$|bc|.|ad| = |ab|.|cf|$

$\Rightarrow$ $|bc|.|ad| = |ac|.|be| = |ab|.|cf|$ Q.E.D.

(c) (i) $\text{Scale Factor} = \dfrac{\text{length of image side}}{\text{length of corresponding object side}} = \dfrac{11 \cdot 2}{4} = 2 \cdot 8$

(ii) $|ab| = \dfrac{7}{2 \cdot 8} = 2 \cdot 5$.

(iii) Area $\Delta ocd = 4 \cdot 5 \times (2 \cdot 8)^2 = 35 \cdot 28$ square units.

5. (a) $\text{Area} = \dfrac{1}{2}(3)(4)\sin 55° = 4 \cdot 9\text{cm}^2$.

(b) (i) $\dfrac{1}{2}|bc| \times 5 = 15$ …multiply both sides by 2

$\Rightarrow$ $5|bc| = 30$ $\Rightarrow$ $|bc| = 6$.

(ii) $\tan|\angle cab| = \dfrac{6}{5}$ $\Rightarrow$ $|\angle cab| = \tan^{-1}\left(\dfrac{6}{5}\right) = 50°$.

(iii) $|\angle bca| = 180 - 90 - 50 = 40°$.

(c) (i) Because Δpqr is isosceles, $|\angle pqr| = |\angle prq| = 70°$

Using the sine rule $\dfrac{|pr|}{\sin 70} = \dfrac{15}{\sin 40}$

$\Rightarrow$ $|pr|\sin 40 = 15 \sin 70$

$\Rightarrow$ $|pr| = \dfrac{15 \sin 70}{\sin 40} = 22$ cm.

(ii) $|\angle prs| = 110°$

Using the cosine rule

$|ps|^2 = 10^2 + 22^2 - 2(10)(22)\cos 110°$

$\Rightarrow$ $|ps| = \sqrt{10^2 + 22^2 - 2(10)(22)\cos 100°}$

$\Rightarrow$ $|ps| = 27$ cm.

6. (a) (i) $P[D]=\frac{1}{6}$

(ii) $P[\text{Vowel}]=\frac{2}{6}=\frac{1}{3}$

(b) (i)

(1,A)	(1,B)	(1,C)
(2,A)	(2,B)	(2,C)
(3,A)	(3,B)	(3,C)
(4,A)	(4,B)	(4,C)

(ii) $P[(2,C)]=\frac{1}{12}$

(iii) $P[(\text{odd, A})]=\frac{2}{12}=\frac{1}{6}$

(iv) $P[\text{C included}]=\frac{4}{12}=\frac{1}{3}$

(c) (i) $5\times4\times3=60$

(ii) $2\times4\times3=24$

(iii) $4\times3\times1=12$

(iv) Numbers which are less than 400 and divisible by 5 are 235 and 325.

$\Rightarrow$ there are two numbers which are in both sets.

7. (a) Firstly, list the numbers in increasing order:

3, 5, 8, 11, 16

$\Rightarrow$ Median is 8.

(b) Graph Required.

(c) (i) $16+12+32+12=72$

(ii) $\text{Mean}=\frac{(10\times16)+(25\times12)+(40\times32)+(65\times12)}{72}=35$

(iii) $16+12+32=60$.

8. (a) (i) $|\angle abc|=55°$.

(ii) $|\angle cda|=180-55=125°$.

(b) Proof in textbook.

(c) $|ak|.|kb|=|ck|.|kd|$ $\quad\ldots$but $|ck|=3|kd|$

$\Rightarrow$ $|ak|.|kb|=3|kd|.|kd|$

$\Rightarrow$ $3|kd|^2=|ak|.|kb|$

$\Rightarrow$ $3|kd|^2=(6)(8)$

$\Rightarrow$ $3|kd|^2=48$

$\Rightarrow$ $|kd|^2=16$

$\Rightarrow$ $|kd|=4$ $\quad\ldots$but $|kd|$ is half the radius

$\Rightarrow$ $r=8$.

9. (a) (i) $\vec{r}$ goes from o to the point directly above x and horizontally across from y.

(ii) $\vec{s}$ goes from o towards b but just two thirds of the way.

(b) (i) $\left|\vec{p}\right| = \sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$.

(ii)
$$
\begin{aligned}
5\vec{p} - \vec{q} &= 5\left(2\vec{i} - \vec{j}\right) - \left(-3\vec{i} + 4\vec{j}\right) \\
&= 10\vec{i} - 5\vec{j} + 3\vec{i} - 4\vec{j} \\
&= 13\vec{i} - 9\vec{j}
\end{aligned}
$$

(iii)
$$
\begin{aligned}
\overrightarrow{pq} = \vec{q} - \vec{p} &= -3\vec{i} + 4\vec{j} - \left(2\vec{i} - \vec{j}\right) \\
&= -3\vec{i} + 4\vec{j} - 2\vec{i} + j \\
&= -5\vec{i} + 5\vec{j}
\end{aligned}
$$

(iv) $\vec{p} \cdot \vec{q} = (2)(-3) + (-1)(4) = -6 - 4 = -10$

(c) $\vec{u} = 2\vec{i} + 5\vec{j}$ $\quad\quad$ $\vec{v} = 8\vec{i} + 10\vec{j}$

(i) $\vec{u} + h\vec{v} = k\vec{i}$

$\Rightarrow$ $2\vec{i} + 5\vec{j} + h\left(8\vec{i} + 10\vec{j}\right) = k\vec{i}$

$\Rightarrow$ $2\vec{i} + 5\vec{j} + 8h\vec{i} + 10h\vec{j} = k\vec{i}$

$\Rightarrow$ $(2+8h)\vec{i} + (5+10h)\vec{j} = k\vec{i}$

$\Rightarrow$ $2 + 8h = k$ and $5 + 10h = 0$

$\Rightarrow$ $10h = -5$

$\Rightarrow$ $h = -\dfrac{1}{2}$

$\Rightarrow$ $2 + 8\left(-\dfrac{1}{2}\right) = k$

$\Rightarrow$ $k = 2 - 4$

$\Rightarrow$ $k = -2$.

(ii) $\vec{u}^{\perp} + h\vec{v}^{\perp} = k\vec{i}^{\perp}$

iff $5\vec{i} - 2\vec{j} - \dfrac{1}{2}\left(10\vec{i} - 8\vec{j}\right) = -2\left(-\vec{j}\right)$

iff $\cancel{5\vec{i}} - 2\vec{j} - \cancel{5\vec{i}} + 4\vec{j} = -2\left(-\vec{j}\right)$

iff $2\vec{j} = 2\vec{j}$ $\quad$ Q.E.D.

10. (a) $2+\frac{2}{5}+\frac{2}{25}+\cdots\cdots$

$a=2, \quad r=\frac{1}{5}$

$S_\infty=\frac{a}{1-r}=\frac{2}{1-\frac{1}{5}}=\frac{10}{5-1}=\frac{10}{4}=2\cdot 5.$

(b) (i) $(1+2x)^3=1+6x+12x^2+8x^3.$

(ii) $(1+2x)^3+(1-2x)^3=2(a+bx^2)$

$\Rightarrow$ $1+\cancel{6x}+12x^2+\cancel{8x^3}+1-\cancel{6x}+12x^2-\cancel{8x^3}=2x+2bx^2$

$\Rightarrow$ $2+24x^2=2a+2bx^2$

$\Rightarrow$ $2a=2$ and $2b=24$

$\Rightarrow$ $a=1$ and $b=12.$

(c) (i) $A=2000(1\cdot 04)^6=€2,531.$

(ii) $A=2000\left[1\cdot 04+1\cdot 04^2+1\cdot 04^3+1\cdot 04^4+1\cdot 04^5+1\cdot 04^6\right]$

$\Rightarrow$ $A=2000\left[\frac{1\cdot 04\left(1-1\cdot 04^6\right)}{1-1\cdot 04}\right]=€13,797$

11. (a) (i) $m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-0}{0+5}=\frac{2}{5}$

$y-y_1=m(x-x_1)$

$\Rightarrow$ $y-0=\frac{2}{5}(x+5)$ …multiply both sides by 5

$\Rightarrow$ $5y=2x+10$

$\Rightarrow$ $2x-5y+10=0$

(ii) I: $2x-5y+10\geq 0$

II: $x\leq 0$

III: $y\geq 0$

(b) (i) I: $3x+2y\leq 60$

II: $5x+2y\leq 80$

(ii) Need to find where the two lines intersect:

$3x+2y=60$

$5x+2y=80$ …subtract

$2x=20$

$\Rightarrow$ $x=10\Rightarrow y=15$

$\Rightarrow$ Lines intersect at $(10,15)$

$\Rightarrow$ Maximum rental income occurs at $(10,15)$ or $(16,0)$ or $(0,30)$

Rental Income $=65x+40y$

Try $(10,15)$: $65(10)+45(15)=650+675=€1,325$

Try $(16,0)$: $65(16)+45(0)=€1,040$

Try $(0,30)$: $65(0)+45(30)=€1,350$

$\Rightarrow$ Developer should include 0 cottages and 30 apartments to maximise potential rental income.

(iii) Construction Costs $=200000x+120000y$

Try $(10,15)$: $200000(10)+120000(15)=€3,800,000$

Try (16,0): $200000(16)+120000(0)=€3,200,000$

Try $(0,30)$: $200000(0)+120000(30)=€3,600,000$

$\Rightarrow$ Developer should include 16 cottages and 0 apartments to minimise construction costs.